Adaptive Reconfigurable Flight Control System Based on Recursive System Identification

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Abstract

An adaptive reconfigurable control algorithm is proposed for aircraft fault tolerant control. An input-output model is derived from a discrete state-space model. The formulated model has the same structure as the ARX model, and therefore any recursive system identification method can be used. Model following control schemes are applied for reconfigurable control system design. The reference outputs for the system to be followed are generated via the linear optimal control theory. During the recursive adaptive control process, the reference system model is updated periodically. The proposed algorithm is very robust and applicable on real time. To validate the proposed adaptive fault tolerant control algorithm, numerical simulation is performed.

1. Introduction

A reconfigurable flight control system(RFCS) is a control system that can accommodate faults by redesigning the control system. A RFCS provides a significant enhancement to flight safety and performance in the event of system fault. Recently, reconfiguration control has been widely studied due to its clear benefits. Previously proposed approaches include model following technique,[1] eigenstructure assignment method,[2] variable structure control scheme,[3] and neural network based adaptive control.[4]

In this study, an input-output model is developed for identification of faulty system. This model has the same structure as ARX(Auto Regressive eXternal) model. Hence, any recursive system identification methods are applicable. We present a recursive identification method that is adequate to the input-output model. And an adaptive reconfigurable control method combined with model following schemes is proposed. An adaptive reconfiguration control with an estimator is also presented. The noise effects can be excluded from the output signals through the estimator. The numerical simulation is performed to validate the proposed control method.

2. Input-output Model and Identification

In this section, we first derive an input-output model from state-space model. This model will be used to describe the real system. Then, a recursive system identification method is developed using input-output signals. The recursive identification method is adequate to identify the proposed input-output model.

A. Input-output model

Consider the discrete-time model of a system in a state-space form

$$x(k+1) = Ax(k) + Bu(k) \qquad (1)$$

$$y(k) = Cx(k) \qquad (2)$$

where $x(k) \in R^n$, $u(k) \in R^m$, $y(k) \in R^p$, and A, B, C are system, input influence, and output influence matrices, respectively. The discrete system matrix A is a state transition matrix, and is always nonsingular. Therefore, the output variables can be described using the state variables and inputs as follows:

$$y(k-j) = CA^{-j}x(k) - \sum_{i=0}^{j-1} CA^{-j+i}Bu(k-i-1) \qquad (3)$$

where $j = 0, \cdots, q-1$. In the matrix form, the above equation can be expressed as

$$Y(k) = Hx(k) + PU(k) \qquad (4)$$

where

$$Y(k) = \begin{bmatrix} y(k) \\ y(k-1) \\ \vdots \\ y(k-q+1) \end{bmatrix}, \quad H = \begin{bmatrix} C \\ CA^{-1} \\ \vdots \\ CA^{-q+1} \end{bmatrix}, \quad U(k) = \begin{bmatrix} u(k-1) \\ u(k-2) \\ \vdots \\ u(k-q+1) \end{bmatrix},$$

$$P = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -CA^{-1}B & 0 & 0 & 0 \\ \vdots & & \ddots & \vdots \\ -CA^{-q+1}B & -CA^{-q+2}B & & -CA^{-1}B \end{bmatrix}$$

Note from Eq. (4) that the state variable $x(k)$ can be obtained from the given input and output signals. If the size of matrix H is $n \times n$ and the rank of matrix A is n, then $x(k)$ can be uniquely determined by using $Y(k)$ and $U(k)$. Using Eq. (1) and Eq. (4), the following equation can be obtained.

$$\begin{aligned} Y(k+1) &= H\,x(k+1) + P\,U(k+1) \\ &= HAH^{+}[Y(k) - P\,U(k)] + HB\,u(k) + PU(k+1) \end{aligned} \qquad (5)$$

where H^{+} is the pseudo-inverse of matrix H. If the rank of matrix H^{+} equals the system order n, then the solution $x(k)$ satisfying Eq. (4) is uniquely determined. However, too many parameters are required to represent the system using Eq. (5).

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Therefore, simple manipulation is performed to reduce the number of parameters.

Using the definition of $U(k)$, the following input history equation can be constructed.

$$U(k+1) = I_m\, u(k) + J_m\, U(k) \tag{6}$$

where

$$I_m = \begin{bmatrix} I_{m\times m} & 0 & \cdots & 0 \end{bmatrix}^T, \quad J_m = \begin{bmatrix} 0 & 0 & 0 & 0 \\ I_{m\times m} & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 \\ 0 & 0 & I_{m\times m} & 0 \end{bmatrix}$$

and $I_{m\times m}$ is an $m\times m$ identity matrix.

Substituting Eq. (6) into Eq. (5) yields

$$\begin{aligned} Y(k+1) &= HAH^+[Y(k) - PU(k)] + \begin{bmatrix} CB\,u(k) \\ 0 \\ \vdots \\ 0 \end{bmatrix} + P\,J_m U(k) \\ &= HAH^+[Y(k) - PU(k)] + I_p CB\,u(k) + P\,J_m U(k) \end{aligned} \tag{7}$$

where $I_p = \begin{bmatrix} I_{p\times p} & 0 & \cdots & 0 \end{bmatrix}^T$, and $I_{p\times p}$ is a $p\times p$ identity matrix. Note from Eq. (7) that the matrix HAH^+ plays the same role as a system matrix. Let us define M_Y and M_U as

$$M_Y H = HA, \quad M_U = PJ_m - M_Y P \tag{8}$$

Then, Eq. (7) can be rewritten as follows.

$$Y(k+1) = M_Y\, Y(k) + M_U\, U(k) + I_p CB\, u(k) \tag{9}$$

Note from Eq. (8) that matrix M_Y has the following structure.

$$M_Y = HAH^+ + U\,Z \tag{10}$$

where $ZH = 0$. The matrix U is a design parameter computed in consideration of the structure of matrix H. If the size of matrix H is $n\times n$, and the rank of matrix H is *n*, then the solution is uniquely determined. By selecting the matrix U appropriately, the matrix M_Y can be constructed in the following form.

$$M_Y = HAH^+ + UZ = J_p + I_p\,\alpha \tag{11}$$

where $\alpha = [\alpha_1 \ \cdots \ \alpha_q]$, and $\alpha_i \in R^{p\times p}$. From Eq. (8) and Eq. (11), M_U is given by

$$\begin{aligned} M_U &= PJ_m - (J_p + I_p\,\alpha)P \\ &= (PJ_m - J_p P) - I_p\,\alpha\,P \end{aligned} \tag{12}$$

In consideration of the structures of P, J_m, and I_p, we have

$$PJ_m - J_p P = 0 \tag{13}$$

Therefore, we have

$$M_U = -I_p\,\alpha\,P \tag{14}$$

Substituting Eqs. (8), (11), and (14) into Eq. (7), we have

$$Y(k+1) = (J_p + I_p\,\alpha)Y(k) - I_p\,\alpha\,P\,U(k) + I_p CB\,u(k) \tag{15}$$

In the consideration of the structure of matrix P, the output variable $y(k+1)$ can be obtained by pre-multiplying $I_p{}^T$ to Eq. (15) as follows:

$$y(k+1) = \alpha Y(k) - \alpha P\,U(k) + CB\,u(k) \tag{16}$$

Equation (16) is used to identify the real system for the adaptive control system. It has the same structure as the ARX model. Therefore, any recursive numerical method for conventional system identification can be used.

B. Recursive Identification Algorithm

Equation (16) can be rewritten as

$$y(k+1) = [\alpha \quad CB \quad -\alpha P] \begin{bmatrix} Y(k) \\ u(k) \\ W(k) \end{bmatrix} = \theta\,\phi(k) \tag{17}$$

where θ is a parameter matrix to be identified. In Eq. (17), the number of parameters to represent α is $p\times pq$, $p\times m$ for CB, and $p\times(q-1)m$ for αP. Therefore, the total number of parameters in Eq. (17) is $p\times(pq+qm)$. If pq is equal to the system order *n*, then the total number of parameters becomes the same as the number of parameters required for system identification.

To identify the parameters, s-sets of data can be used. The chosen data sets satisfy the following equation.

$$\begin{aligned} Y &= \begin{bmatrix} y(k+1) & y(k+2) & \cdots & y(k+s) \end{bmatrix} \\ &= \theta\,[\phi(k) \quad \phi(k+1) \quad \cdots \quad \phi(k+s-1)\,] \\ &= \theta\,\Phi \end{aligned} \tag{18}$$

Equation (18) consists of $p\times s$ constraint equations, and the number of parameters is $p\times(p+m)q$. To obtain the better solution, s should be greater than $(p+m)q$. If not, there exists a uniqueness problem, and the identified model may not represent the real system appropriately.

Consider the system that has the time-varying parameters. In this case, there may exist an estimation error between the identified parameter matrix $\theta(k-1)$ in the $(k-1)$ step and the current parameter matrix $\theta(k)$; that is

$$Y - \theta(k-1)\Phi = \delta\theta\,\Phi \tag{19}$$

where $\delta\theta = \theta(k) - \theta(k-1)$. The least square solution of $\delta\theta$ can be obtained as follows.

$$\delta\theta = [Y - \theta(k-1)\Phi]\,\Phi^T[\Phi\Phi^T]^{-1} \tag{20}$$

By using Eq. (20), the current parameter matrix can be updated by using the following recursive equation.

$$\begin{aligned} \theta(k) &= \theta(k-1) + \kappa\,\delta\theta \\ &= \theta(k-1) + \kappa\,[Y - \theta(k-1)\Phi]\,\Phi^T[\Phi\Phi^T]^{-1} \end{aligned} \tag{21}$$

where the positive scalar $\kappa < 1$ is the step-size for the parameter identification.

3. Adaptive Control System Design

A mathematical model for an adaptive control system is usually obtained by on-line system identification based on input-output signals. The on-line recursive identification method uses the limited input signals to excite the system in a finite time. For this reason, the identified model by the on-line recursive identification method is less accurate than that obtained by the off-line identification method. Therefore, in this paper, the model following control scheme is adopted to compensate the effects of the identification errors.

A. Adaptive Model Following Controller without Estimator

In this section, the key procedures to design the adaptive model following controller are described as follows.

Step 1. Determine the system model in the form of Eq. (16). The initial model can be identified using input-output signals or obtained by mathematical modeling methods. During the recursive adaptive control process, however, the system model is updated periodically via the on-line identification method addressed in Sec. 2.B.

Step 2. Generate reference outputs for the system to be followed. To design a stabilizing control law for the system model, Eq. (16), let us rewrite Eq. (6) and Eq. (15) in matrix form as follows.

$$\begin{bmatrix} Y(k+1) \\ U(k+1) \end{bmatrix} = \begin{bmatrix} J_p + I_p\alpha & -I_p\alpha P \\ 0 & J_m \end{bmatrix} \begin{bmatrix} Y(k) \\ U(k) \end{bmatrix} + \begin{bmatrix} I_p CB \\ I_m \end{bmatrix} u(k) \quad (22)$$

Note that the above equation has the same form as state-space model. Therefore, a control law stabilizing Eq. (22) can be easily designed as follows:

$$u^*(k) = -K_1\, Y(k) - K_2\, U(k) = -K \begin{bmatrix} Y(k) \\ U(k) \end{bmatrix} \quad (23)$$

The gain matrices of the above controller can be determined by applying LQR theory. It is obvious from Eq. (4) that the above control law also stabilizes $y(k+1)$. Therefore, substituting Eq. (23) into Eq. (16) yields the desired model responses as

$$y^*(k+1) = \alpha\, Y(k) - \alpha\, P\, U(k) + CB\; u^*(k) \quad (24)$$

Step 3. Design the model following controller. The objective of the model following control scheme is to make the system output follow the desired responses; that is

$$\begin{aligned} y^*(k+1) &= y(k+1) \\ &= \alpha\, Y(k) - \alpha\, P\, U(k) + CB\; u(k) \end{aligned} \quad (25)$$

The control input can be determined using the pseudo-inverse as follows

$$u(k) = (CB)^+ (y^*(k+1) - \alpha\, Y(k) + \alpha\, P\, U(k)) \quad (26)$$

Note that the existence of $u(k)$ is dependent on the structure of matrix CB. When the number of outputs is greater than the number of inputs, the model following can be achieved in the least square sense, thereby the control input can be over-designed. On the other hand, when the number of outputs is less than the number of inputs, there exist several solutions for the input to make the system outputs follow the desired outputs. In this case, the control inputs can be designed such that the magnitude of control inputs may be minimized. Consider the following objective function.

$$\begin{aligned} J &= [y^*(k+1) - y(k+1)]^T Q\, [y^*(k+1) - y(k+1)] \\ &\quad + u^T(k) R\, u(k) \end{aligned} \quad (27)$$

where Q and R are positive definite weighting matrices. The first term is related to the model following performance, and the second term is related to the control energy minimization. By applying the optimality condition, the control input can be obtained as

$$u(k) = G[y^*(k+1) - \alpha\, Y(k) + \alpha\, PU(k)] \quad (28)$$

where $G = [R + B^T C^T QCB]^{-1} B^T C^T Q$.

Step 4. Go to 1 or 2. The model following control procedure is iterated at every time step. However, the system model is updated periodically at $n\,(>1)$ time steps instead of every time step to reduce the computation time.

B. Adaptive Model Following Controller with Estimator

In general, the output signals include some measurement noise. To accommodate the noise effects from the output signals, an estimator can be used. The estimator for the system shown in Eq. (15) can be constructed as follows:

$$\begin{aligned} \hat{Y}(k+1) &= (J_p + I_p\,\alpha)\hat{Y}(k) - I_p\,\alpha PU(k) + I_p CB\, u(k) \\ &\quad + L\,(Y(k) - \hat{Y}(k)) \end{aligned} \quad (29)$$

Let us introduce the estimation error, $E(k) = Y(k) - \hat{Y}(k)$. Then, the estimator error equation can be obtained by using Eq. (15) and Eq. (29) as

$$E(k+1) = (J_p + I_p\alpha - L)E(k) \quad (30)$$

Assume that the estimator gain L_0 stabilizes the system matrix $(J_p + I_p\alpha_0)$ where α_0 is an identified parameter matrix using the initial model, and L_0 is the estimator gain matrix designed by using the initial model. Then, the state estimator gain L at the current step can be designed as

$$L = I_p\alpha - I_p\alpha_0 + L_0 \quad (31)$$

The above equation provides an efficient algorithm for designing the estimator gain matrix.

Consider the following objective function.

$$\begin{aligned} J &= [y^*(k+1) - \bar{y}(k+1)]^T Q\, [y^*(k+1) - \bar{y}(k+1)] \\ &\quad + u^T(k) R\, u(k) \end{aligned} \quad (32)$$

where the estimated output $\bar{y}(k+1)$ is defined as follows:

$$\bar{y}(k+1) = \alpha\, \hat{Y}(k) - \alpha\, PU(k) + CB\; u(k) \quad (33)$$

Note from the above equation that the estimated output from Eq. (29) is used instead of using the identified model, Eq. (15). Substituting Eq. (33) into Eq. (32), and by applying the optimality condition to the resulting equation, the control input can be obtained as follows

$$u(k) = G[y^*(k+1) - \alpha\hat{Y}(k) + \alpha\, PU(k)] \quad (34)$$

where $G = [R + B^T C^T QCB]^{-1} B^T C^T Q$.

4. Numerical Examples

In this section, a numerical example is presented to verify the adaptive RFCS developed in the Sec. 3. For a damaged system model, a high performance aircraft with a critical damage on the wing surface is considered. Proper orthogonal decomposition (POD) technique is applied to reduce the size of the system matrix down to three. Reduced order system matrices can then be used to predict the initial parameter quickly and accurately in the adaptive reconfigurable control system. Figures 1 and 2 show the sensor signal of the open loop and closed loop system for the critical damage model. The control input histories of the closed loop system is shown in Figure 3. Figures show that after a short period of time for system identification, the developed reconfigurable controller suppresses the aeroelastic instability.

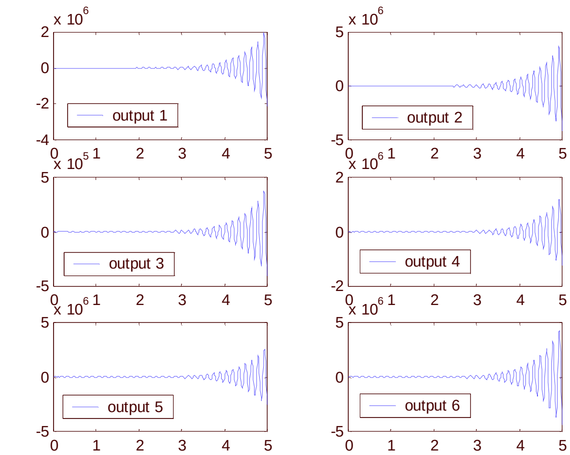

Figure 1. Open loop sensor signals of the damaged model

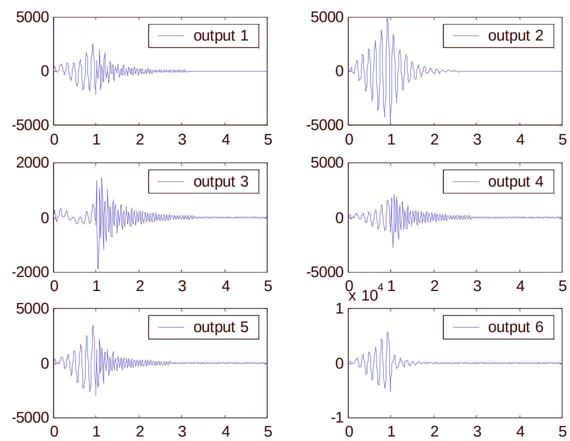

Figure 2. Closed loop sensor signals of the damaged model

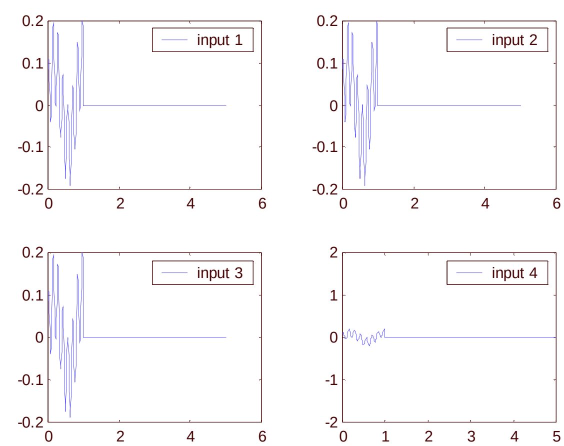

Figure 3. Control input histories

Conclusions

An adaptive reconfigurable control algorithm is proposed for aircraft fault tolerant control. An input-output model is developed to describe the real system. This model has the same structure as an ARX model that can be easily identified by recursive algorithms. To identify the system model, a recursive method is presented that is adequate to the proposed input-output model. For the control system, adaptive control schemes are proposed with the combination of model following control scheme. The reference outputs for the system to be followed were generated using the LQR theory. Also, an adaptive reconfiguration control with an estimator is presented. The proposed reconfigurable control is very robust and applicable on real time. To demonstrate the proposed adaptive fault tolerant control algorithm, numerical simulation is performed.

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